In the name of God

Department of Aerospace Engineering

Automatic Control
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Spring 2024
Homework 2
Due: Mar 15, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner.
- You may use MATLAB and Simulink only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn to solve the problems by hand to understand how the method works and improve your problem-solving acumen.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format AC_HW1_StudentNumber on the Sharif Courseware (CW). The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Consider the double-pendulum system as shown in Figure 1. In this system two pendulums suspended from frictionless pivots and connected at their midpoints by a spring. Assume that each pendulum can be represented by a mass M at the end of a massless bar of length L. The spring located in the middle of the bars is unstretched when $\theta_1 = \theta_2$. The input force is represented by $f(t)$, which influences the left-hand bar only.

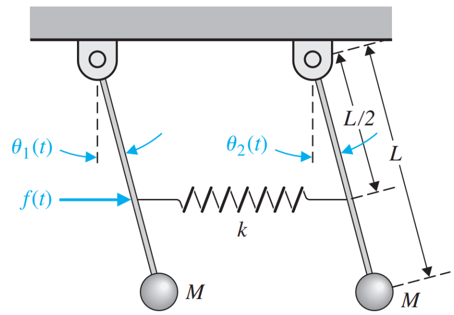

Figure 1: Double-pendulum system

   (a) Obtain the equations of motion.
   (b) Assume that the displacement is small and use linear approximations $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$ to find a linear model for the system in state space form.
   (c) Determine the transfer function $T(s) = \frac{\theta_1(s)}{F(s)}$. You may obtain the transfer function by taking the Laplace transform of equations and eliminating θ_2 or you can use $T(s) = C(sI - A)^{-1}B + D$. Assume $a = \frac{g}{L} + \frac{k}{4M}$, $b = \frac{k}{4M}$ and $c = \frac{1}{2ML}$ for simplicity.

2. Represent the rotational mechanical system shown in Figure 2 in state space where the input and output are T and θ_1, respectively. (Hint: Find the equivalent system by eliminating the gear using $\frac{T_2}{T_1} = \frac{N_2}{N_1}$.)

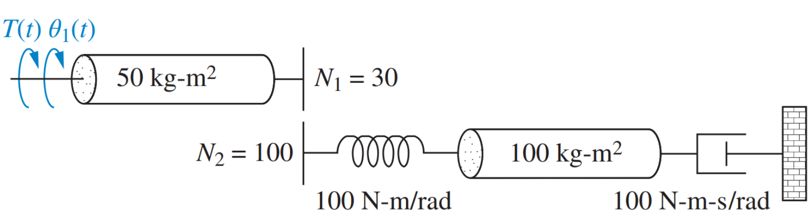

Figure 2: Rotational mechanical system

3. A plane (two-dimensional) model of an overhead crane and its load is depicted in Figure 3, where x, θ and l are the trolley position, swing angle, and hoisting rope length (variable), respectively. The trolley is driven by a force f_x, and load is suspended from the trolley by a rigid rope and driven by a force f_L. Assume that the mass of the crane is divided into m_x (traveling) and m_l (hoisting down) components, which include the equivalent masses of the rotating parts such as motors and their drive trains. In addition d_{vx} and d_{vl} denote the viscous damping coefficients associated with the x and l motions, respectively.

(a) Obtain equations of motion for overhead crane system using Lagrange's equation.

(b) (Bonus) Simulate the initial condition response of the (nonlinear) system to the $x(0) = 1$, $\dot{x}(0) = 0$, $\theta(0) = 0.3\ rad$, $\dot{\theta}(0) = 0$, $l(0) = 10\ m$ and $\dot{l}(0) = 0$ initial conditions in Simulink. The overhead crane has the following parameters: $m = 300\ kg$, $m_x = 1000\ kg$, $m_l = 500\ kg$, $d_{vx} = 250\ kg/s$ and $d_{vl} = 200\ kg/s$. [**M**]

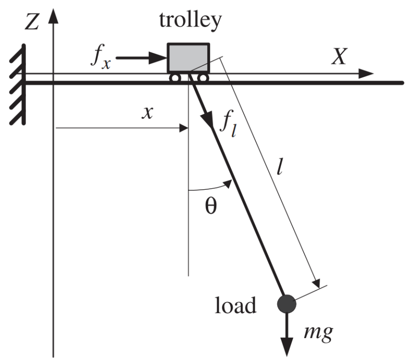

Figure 3: Plane model of an overhead crane system

4. A very typical problem of electromechanical position control is an electric motor driving a load that has one dominant vibration mode. The problem arises in computer-disk-head control, reel-to-reel tape drives, and many other applications. A schematic diagram is sketched in Figure 4. The motor has an electrical constant (back-emf) K_e, a torque constant K_t, an armature inductance L_a, and a resistance R_a. The rotor has an inertia J_1 and a viscous friction B. The load has an inertia J_2. The two inertias are connected by a shaft with a spring constant k and an equivalent viscous damping b. Write the equations of motion.

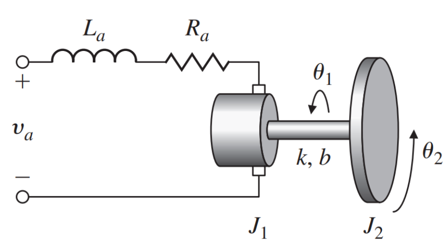

Figure 4: Motor with a flexible load

5. Consider the electrical circuit that is shown in Figure 5.

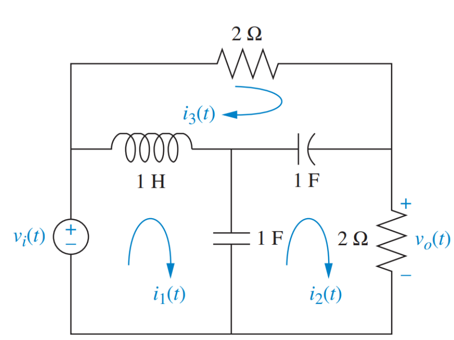

Figure 5: Electrical circuit

(a) Obtain the differential equations describing the system.

(b) Find the state-space representation of the system if the output is $v_o(t)$.

(c) Find the transfer function of the system from the state-space model.

6. Consider the liquid-level control system shown in Figure 6. The inlet valve is controlled by a hydraulic integral controller. Assume that the steady-state inflow and outflow rates are $\bar{Q}$, the steady-state head is $\bar{H}$, steady-state pilot valve displacement is $\bar{X} = 0$, and steady-state valve position is $\bar{Y}$. We assume that the fixed set-point corresponds to the steady-state head $\bar{H}$. Assume also that the disturbance inflow rate q_d, which is a small quantity, is applied to the water tank at $t = 0$. This disturbance causes the head to change from $\bar{H}$ to $\bar{H}+h$. This change results in a change in the outflow rate by q_o. Through the hydraulic controller, the change in head causes a change in the inflow rate from $\bar{Q}$ to $\bar{Q} + q_i$. (The integral controller tends to keep the head constant as much as possible in the presence of disturbances.) We assume that all changes are of small quantities. We assume that the velocity of the power piston (valve) is proportional to pilot-valve displacement x ($dy/dt = K_1 x$), where K_1 is a positive constant. We also assume that the change in the inflow rate q_i is negatively proportional to the change in the valve opening y ($q_i = -k_v y$) where K_v is a positive constant. Assuming the following numerical values for the system, $C = 2\ m^2$, $R = 0.5\ sec/m^2$, $K_v = 1\ m^2/sec$, $a = 0.25\ m$, $b = 0.75\ m$, $K_1 = 4\ sec^{-1}$ obtain the transfer function $\frac{H(s)}{Q_d(s)}$.

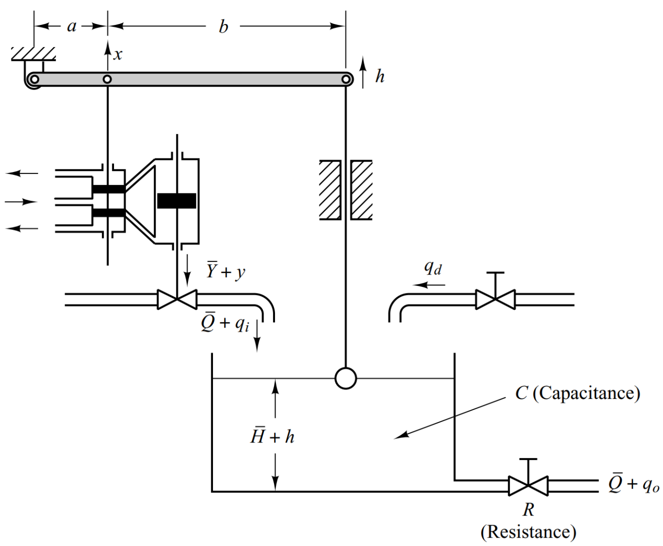

Figure 6: Liquid-level control system

7. Consider a communication satellite of mass m orbiting around the earth. The position of the satellite is specified by r, θ and ϕ as shown below:

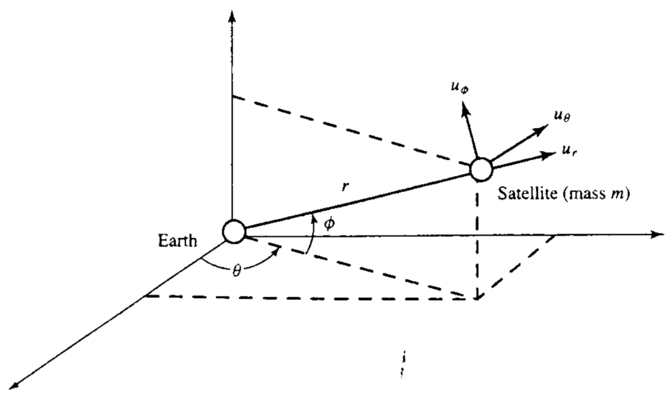

Figure 7: Schematic of communication satellite orbiting

The orbit can be controlled by three orthogonal thrusts u_r, u_θ and u_ϕ. The system can be shown to be described by the following nonlinear equations:

$$\begin{cases} \ddot{r} = r\dot{\theta}^2\cos^2(\phi) + r\dot{\phi}^2 - \dfrac{k}{r^2} + \dfrac{u_r}{m} \\ \ddot{\theta} = -2\dfrac{\dot{r}\dot{\theta}}{r} + 2\dot{\theta}\dot{\phi}\tan(\phi) + \dfrac{u_\theta}{mr\cos(\phi)} \\ \ddot{\phi} = -\dot{\theta}^2\cos(\phi)\sin(\phi) - 2\dfrac{\dot{r}\dot{\phi}}{r} + \dfrac{u_\phi}{mr} \end{cases}$$

Consider $x = \begin{bmatrix} r & \dot{r} & \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^T$, $u = \begin{bmatrix} u_r & u_\theta & u_\phi \end{bmatrix}^T$ and $y = \begin{bmatrix} r & \theta & \phi \end{bmatrix}^T$ as state vector, input vector and output vector respectively. In addition, $k = r_o^3\omega_o^2$ is a known constant.

(a) Show that this system has an equilibrium path as $u_e(t) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $x_e(t) = \begin{bmatrix} r_o & 0 & \omega_o t & \omega_o & 0 & 0 \end{bmatrix}^T$ assuming that the satellite is orbiting in a circular equatorial orbit. (Hint: Some systems can have an equilibrium path $(x_e(t), u_e(t))$ instead of the equilibrium point (x_e, u_e) which is defined as $\dot{x}_e(t) = f(x_e(t), u_e(t), t)$. To solve this part, use $\dot{x}_e(t) = \begin{bmatrix} 0 & 0 & \omega_0 & 0 & 0 & 0 \end{bmatrix}^T$ which is obtained based on the physical behavior and motion of the system at this equilibrium path.)

(b) Linearize the system around the aforementioned equilibrium path and obtain the state space representation of the system.

(c) Linearize the system and obtain the state space model of the system assuming $m = 10\ kg$, $r_o = 1000\ km$ and $\omega_o = 0.05\ rad/s$ utilizing MATLAB. Compare the result with (b). [**M**]

(d) Obtain the transfer matrix (transfer functions) of the system and determine the transfer function of pitch thruster to pitch angle $\frac{\Theta(s)}{U_\theta(s)}$. [**M**]

Good Luck
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