In the name of God

Department of Aerospace Engineering

Automatic Control
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Spring 2024
Homework 3
Due: Apr 10, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner.

- You may use MATLAB and Simulink only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first.* You will not gain any intuition by looking at results — you need to learn to solve the problems by hand to understand how the method works and improve your problem-solving acumen.

- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.

- Submit a compressed file with the naming format AC_HW1_StudentNumber on the Sharif Courseware (CW). The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.

- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.

- The homework assignments are meant to be completed *individually.* While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.

- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.

- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Aircraft control surface actuators (servomotors), can be modeled as second-order systems. To identify the closed-loop transfer function of the elevator actuator, an experiment is conducted, and the elevator response (deflection) to a step input with a value of $\delta_{e_c} = 0.175$ rad is obtained. The commanded deflection, measured deflection, and estimated deflection for this problem are depicted in Figure 1.

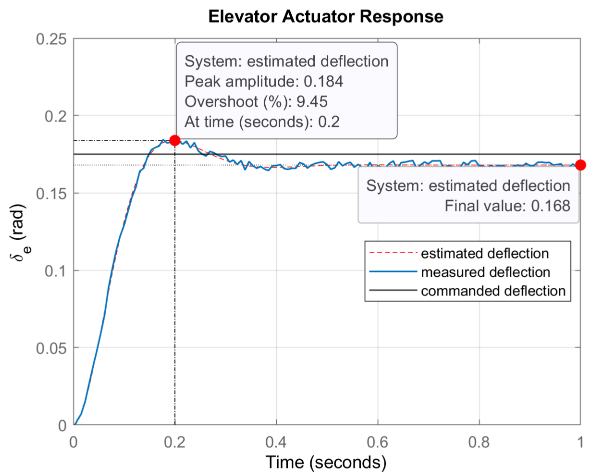

Figure 1: Elevator actuator response

Find the parameters ζ, ω_n, and k using the information provided in the response curve, given that the closed-loop transfer function has the following form:

$$\frac{\delta_e(s)}{\delta_{e_c}(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2. Assume you wish to control the elevation of the satellite-tracking antenna shown in Figure 2. The antenna and drive parts have a moment of inertia J and a damping B; these arise to some extent from bearing and aerodynamic friction, but mostly from the back emf of the DC drive motor. The equation of motion is $J\ddot{\theta} + B\dot{\theta} = T_c$, where T_c is the torque from the drive motor. Assume that $J = 600,000$ kg.m^2, $B = 20,000$ N.m.s.

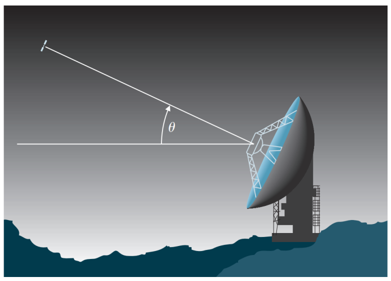

Figure 2: Schematic of satellite-tracking antenna

(a) Find the transfer function between the applied torque T_c and the antenna angle θ.

(b) Suppose the applied torque is computed so that θ tracks a reference command θ_r according to the feedback law

$$T_c = K(\theta_r - \theta),$$

where K is the feedback gain. Find the transfer function between θ_r and θ.

(c) What is the maximum value of K that can be used if you wish to have an overshoot $M_p < 10\%$?

(d) Use MATLAB to plot (`subplot` command) the step response of the antenna system (`step` command) for $K = 200, 400, 1000$, and 2000. Find the overshoot of the four step responses by examining your plots. Do the plots to confirm your calculations in parts (c)? [**M**]

3. Linearizing the nonlinear dynamics of a quadrotor yields the transfer function for its pitch channel $\frac{\theta(s)}{\tau_\theta(s)} = \frac{10}{s^2}$ where θ represents the pitch angle, and τ_θ denotes the pitch control torque about the body y-axis. The block diagram that is depicted in Figure 3 shows a cascade control system to control the pitch angle of the quadrotor.

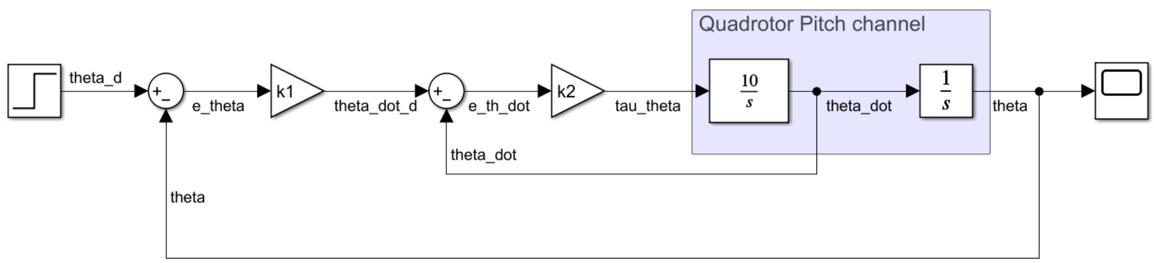

Figure 3: Cascade control system for quadrotor pitch attitude control

(a) To gain familiarity with applying transient response parameters in controller design, find the values of k_1 and k_2 that achieve a closed-loop system with a 10% overshoot and settling time of 5 seconds. (Hint: $T_s = \frac{4}{\zeta\omega_n}$.)

(b) Verify your design by checking the transient repsponse parameters of the closed-loop system using `stepinfo` in MATLAB. [**M**]

(c) Validate your design in part a by simulating the control system in Simulink. Set the step value to 0.1. [**M**]

4. In typical conventional aircraft, longitudinal flight model linearization results in transfer functions with two pairs of complex conjugate poles. Consequently, the natural response for these airplanes has two modes in their natural response. The *short period* mode is relatively well-damped and has a high-frequency oscillation. The *plugoid mode* is lightly damped and its oscillation frequency is relatively low. For example, in a specific aircraft the transfer function from elevator deflection to pitch angle is

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-26.12(s + 0.0098)(s + 1.371)}{(s^2 + 0.009s + 0.004)(s^2 + 4.21s + 18.23)}.$$

(a) Find which of the poles correspond to the short period mode and which to the phugoid.

(b) Peform a *phugoid approximation* (dominant-pole approximation), retaining the two poles and the zero closest to the ω-axis. (Hint: Ensure that the DC gain of the approximate system matches that of the original system. Once you have chosen the dominant poles and zero, determine the appropriate gain to maintain the DC gain unchanged.)

(c) Use MATLAB to compare the step responses of the original transfer function and the approximation. [**M**]

5. A measure of the degree of instability in an unstable aircraft response, for determining its level of flying quality is the amount of time it takes for the *amplitude* of the time response to double, given some nonzero initial condition. If the pole of a first-order system is positive or the damping ratio of a second-order system is negative (unstable system), the time-to-double amplitude will be positive. Conversely, if the system is stable, the time-to-double amplitude will be negative, indicating the time it takes for the amplitude to halve (time-to-halve).

   (a) For a first-order system (like spiral mode), show that the time-to-double is $\tau_2 = \frac{\ln 2}{p}$, where $s = p$ is the ploe of the system. (Hint: consider the response of the first-order system as $y(t) = ke^{pt}$.)

   (b) For a second-order system with complex poles (like phugoid mode), show that $\tau_2 = \frac{\ln 2}{-\zeta\omega_n}$. (Hint: consider the response of the second-order system as $y(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \theta)$)

6. Determine the stability of the closed-loop system with the following characteristic equation using Routh's stability criterion.

$$\Delta(s) = s^6 + 2s^5 + 5s^4 + 15s^3 + 14s^2 + 13s + 10$$

   Find how many poles are in the right half-plane, in the left half-plane, and on the $j\omega$-axis. You may check your answer utilizing `roots` in MATLAB.

7. Consider a unity negative feedback system with the open loop transfer function

$$G(s) = \frac{k}{(s-1)(s+3)(s+5)}.$$

   (a) Find the range of k using Routh's stability criterion for stability of the closed-loop system.

   (b) Find the value of k for marginal stability and pure oscilations in the response. (Hint: simple poles on the $j\omega$-axis)

8. (Bonus) The transfer function relating the output engine fan speed (rpm) to the input main burner fuel flow rate (lb/h) in a short takeoff and landing (STOL) fighter aircraft, ignoring the coupling between engine fan speed and the pitch control command, is

$$G(s) = \frac{1.3s^7 + 90.5s^6 + 1970s^5 + 15000s^4 + 3120s^3 - 41300s^2 - 5000s - 1840}{s^8 + 103s^7 + 1180s^6 + 4040s^5 + 2150s^4 - 8960s^3 - 10600s^2 - 1550s - 415}$$

   (a) Find how many poles are in the right half-plane, in the left half-plane, and on the $j\omega$-axis using Routh's stability criterion.

   (b) Determine poles of the system and its stability by finding roots of characteristics equation in MATLAB (`roots`). [**M**]

   (c) Develop a MATLAB program that prompts the user to input the coefficients of the characteristic equation. The program then utilizes Routh's method to check the stability of the system and displays the result in the output. Verify your program for the given system. [**M**]

Good Luck
M. Shahrajabian