In the name of God

Automatic Control
Dr. S. A. Emami
Spring 2024
Homework 4
Due: May 9, 2024

Department of Aerospace Engineering

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner.
- You may use MATLAB and Simulink only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn to solve the problems by hand to understand how the method works and improve your problem-solving acumen.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format AC_HW1_StudentNumber on the Sharif Courseware (CW). The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Consider the control system that is shown in Figure 1.

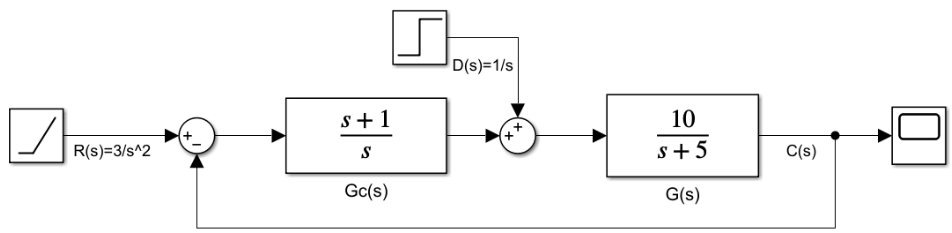

Figure 1: Reference tracking with disturbance rejection control system

   (a) Find the total steady-state error due to a ramp input with a slope of 3 and a unit step disturbance.

   (b) Find the value of m in the new controller, given by $G_c(s) = \frac{s+1}{s^m}$, such that the total steady-state error, accounting for both reference tracking and disturbance rejection, is reduced to zero.

   (c) Simulate the control system in Simulink to validate your answers in part a and part b. [**M**]

   (d) (Bonus) Can you suggest a controller that ensures zero steady-state error for the cosine input $r(t) = \cos(2t)$? How about disturbance rejection alongside cosine reference tracking? Validate your answer using Simulink. [**M**]

2. Sketch the root locus for the given control system with respect to $K_g > 0$ by manually obtaining all the necessary information through the root locus procedure. (Hint: rewrite the characteristic equation as $1 + K_g G'(s) = 0$ and follow the root locus procedure with $G'(s) = 0$)

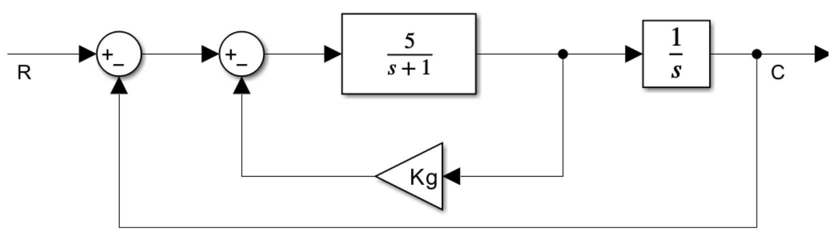

Figure 2: Closed-loop control system

3. Sketch the root locus for the given control system with respect to $K > 0$ by manually obtaining all the necessary information through the root locus procedure.

$$G(s) = \frac{K(5 - s)}{(s + 1)^4}, \quad H(s) = 1.$$

4. Sketch the root locus for the following control system with respect to $K < 0$ by manually obtaining all the necessary information through the root locus procedure.

$$G(s) = \frac{K(s + 0.5)(s - 1)}{s^2(s + 3)}, \quad H(s) = 1.$$

5. In order to analyze the bank angle controllability of a business jet airplane with the pilot in the loop at a cruise altitude of 40,000 feet, a block diagram illustrating the pilot-plus-airplane system (as a closed-loop control system!) is shown in Figure 3. The dynamics of the pilot as a controller (ignoring neuro-muscular lag) can be modeled as

$$G_p(s) = K_p \frac{T_1 s + 1}{T_2 s + 1} e^{-\tau s}.$$

Here, K_p represents the pilot gain, T_1 and T_2 are parameters of the pilot equalization characteristic (compensation), and τ denotes the pilot's reaction time delay (pure time delay between the time error changes on the indicator and the time the command reaches the muscles). The pure time delay $e^{-\tau s}$ can be approximated using the first-order Pade approximation:

$$e^{-\tau s} \approx \frac{1 - \tau/2s}{1 + \tau/2s}$$

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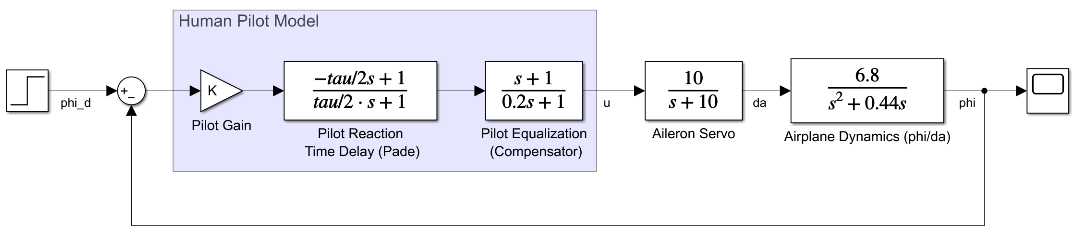

Figure 3: Pilot plus airplane closed-loop control system

(a) Utilize the MATLAB Symbolic Math Toolbox to find the characteristic equation of the closed-loop system based on τ, given that $K_p = 0.2$, $T_1 = 1$ and $T_2 = 0.2$. (Hint: Use the `numden` function to separate numerator and denominator of the closed-loop tansfer function.) [**M**]

(b) What is the maximum allowable pilot time delay (τ) for the closed-loop system (roll channel of airplane) to remain stable? Comment on the results. (Hint: plot the root locus with respect to τ) [**M**]

(c) Verifiy your answer numerically by implementing a `for` loop with τ values ranging from 0.01 to 1, with a step size of 0.0001. Use an if condition to break the loop when one of the roots is too close to the $j\omega$-axis, indicating nearing the stability margin. [**M**]

(d) Validate your answer by simulating the control system in Simulink with $\tau = 0.1$ and $\tau = 0.8$. Set the value of step input to $\phi_d = 0.1$ rad. [**M**]

(e) If $\tau = 0.1$ seconds, plot the root locus for the pilot-plus-airplane system with respect to pilot gain K_p and determine the range of K_p for the stability of airplane (in the roll channel). [**M**]

(f) Using the root loci, determine the value of K_p such that the damping ratio of the dominant complex poles is $\zeta = 0.68$. Simulate the closed-loop control system with the obtained K_p and analyze the result. [**M**]

Good Luck
M. Shahrajabian