In the name of God

Department of Aerospace Engineering

Automatic Control
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Spring 2024
Homework 5
Due: June 9, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner.
- You may use MATLAB and Simulink only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn to solve the problems by hand to understand how the method works and improve your problem-solving acumen.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format AC_HW1_StudentNumber on the Sharif Courseware (CW). The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Suggest the appropriate type of controller (PI, PD, PID, Lead, Lag, Lag-Lead) to stabilize the system (if required), achieve an acceptable transient response, and reduce the steady-state error in response to a step input for the following open-loop systems. You are not required to design the controller, simply explain your choice and justification for it.

   (a) $G_1(s) = \dfrac{1}{s^2 + 0.5s + 0.5}$

   (b) $G_2(s) = \dfrac{1}{s^2 + 40s + 800}$

   (c) $G_3(s) = \dfrac{s+1}{s(s-1)}$

2. Consider the following open loop system

$$G(s) = \frac{1}{s^2 - s - 2}.$$

   Design a Lead compensator for the given open-loop system such that the closed-loop system with negative unity feedback has overshoot less than 16.3% and settling time less than 2 seconds.

   (a) In the first design, consider placing the compensator zero at $s = -z = -1$ to cancel out the stable pole of the open-loop system.

   (b) In the second design, consider placing the compensator zero at the real part of the desired poles.

   (c) Validate your design using the `stepinfo` command and compare the results of parts a and b. [**M**]

3. Consider the following open loop system

$$G(s) = \frac{10}{s(s+2)(s+8)}.$$

   (a) Design a Lag-Lead compensator for the given open-loop system such that the closed-loop system with negative unity feedback has dominant closed-loop poles at $s = -2 \pm j2\sqrt{3}$ and the velocity error constant is set to $k_v = 60$.

   (b) Validate your design by simulating the closed-loop system in Simulink. [**M**]

4. Consider the following open loop system

$$G(s) = \frac{10(s^2 + 2s + 49)}{s(s^2 + 3s + 2)}.$$

   (a) Sketch the Bode plot for the open loop system by manually sketching and summing up the magnitude and phase of all the components of the transfer function.

   (b) Utilize MATLAB to plot the Bode diagram using the `bode` command and compare your answer with part a. [**M**]

5. Frequency response analysis is a practical method used for system identification to experimentally determine an equivalent linear model for the system. When dealing with simple low-order systems and provided with a Bode plot illustrating the system response, one can determine the zero-pole-gain transfer function model by analyzing key features such as the magnitude at the initial frequency, the initial slope, changes in slope of magnitude (breakpoints), as well as the initial and final phase. Analyze the Bode plot shown in the figure to determine the zero-pole-gain transfer function model for the system.

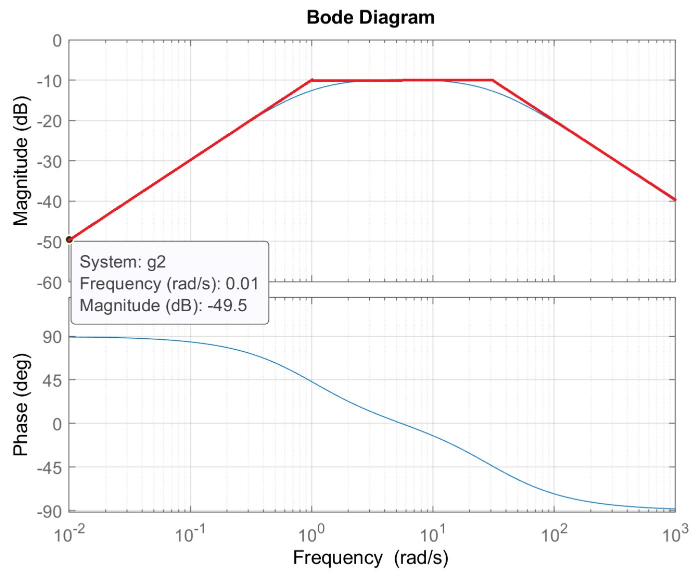

Figure 1: Frequency response of a system illustrated in magnitude and phase (Bode plot)

6. Consider the closed-loop control system shown in Figure 2.

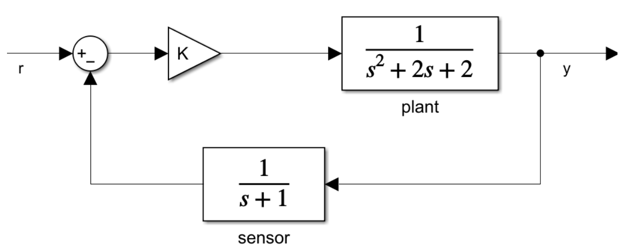

Figure 2: Control system

(a) Draw the Nyquist plot for the system (by hand).

(b) Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K.

7. Consider the following open loop system

$$G(s) = \frac{k}{s(s^2+s+4)}.$$

(a) Find the value of k such that the phase margin of the system is 50 degrees.

(b) What is the gain margin with this k? Is the system stable?

(c) Verify your answer utilizing the `margin` command in MATLAB. [**M**]

8. New advancements in material technology and robotics, combined with the increasing demand of lightweight and portable robotic arms, have driven research into the design and control of flexible robotic manipulators. Flexibility comes from the use of plastic or carbon-fiber frames, which significantly reduces the cost of manufacturing and the power consumption of the system. But with flexibility come a varying degree of inaccuracies in the robotic arm such as increased settling time, especially at the end point. Figure 3 illustrates a hardware platform with a flexible joint module developed by Quanser. The transfer function of the system is

$$G(s) = \frac{\theta(s)}{V_m(s)} = \frac{61.07s^2 + 40126}{s^4 + 34.69s^3 + 1251.6s^2 + 22795.3s}$$

where θ is the servo load gear angle and V_m is the input voltage applied to the servomotor.

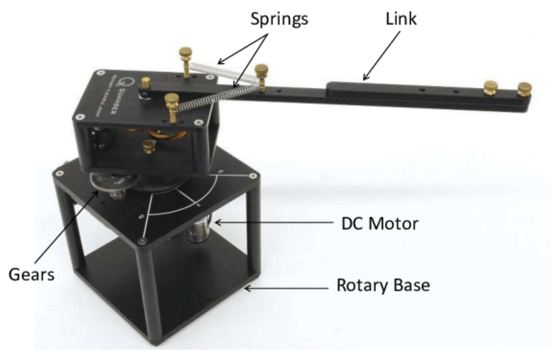

Figure 3: Flexible joint robotic arm by Quanser

(a) Design a lead compensator in frequency domain such that the compensated system has the desired phase margin of $PM_d = 70$ deg and velocity error constant of $K_v = 10$. You may utilize MATLAB to sketch the Bode plot and find the phase margin of the system. Then, verify your design by checking the phase margin of the compensated system using `margin` command. Then, Plot the step response of the closed-loop system in MATLAB and check the transient response characteristics of the closed-loop system. (Hint: You may use `feedback` command in MATLAB to find the closed-loop system) [**M**]

(b) Design a lag-lead compensator using the Control System Designer app in MATLAB to achive a closed-loop system with damping ratio of $\zeta = 0.8$ and bandwidth of $\omega_b = 5$ rad/s. Verify your design by checking the phase margin of the compensated system. Then, validate your design by simulating the closed-loop system in Simulink. (Hint1: Use the approximation $PM \approx 100\zeta$ to find the corresponding desired phase margin, Hint2: Add a lead and a lag compensator in the C block section and then use the Loop Shaping tuning method with the fixed structure option) [**M**]

9. (Bonus) To lower the workload of pilots, particularly on long flights, most airplanes are equipped with some form of Automatic Flight Control System (AFCS) or autopilot system. Heading angle hold is an important mode in many autopilot systems. Heading changes of an aircraft (bank-to-turn vehicles) are executed by roll control. As the lift vector is banked, a

horizontal force component generates a lateral acceleration that turns the velocity vector horizontally. Direct sideslip control is ineffective because of the small lateral projected area of the aircraft; also, the ensuing adverse yaw-roll coupling would be undesirable. In most airplanes heading angle is controlled by establishing a certain bank angle (around the longitudinal axis of body coordinate system) and holding that angle until the desired heading change has been achived. Therefore, the bank angle control loop is often used as an inner loop in a heading angle control system. The bank-angle-to-aileron transfer function for a business jet airplane at a cruise flight condition is as follows:

$$G(s) = \frac{\phi(s)}{\delta_a(s)} = \frac{6.8}{s^2 + 0.44s}$$

(a) A block diagram for the bank angle control system (bank angle hold autopilot) is shown in Figure 4. Find the k_ϕ gain using the root locus method to achieve a closed-loop system with an overshoot of 15%. Then, simulate the control system in Simulink by setting the step command to $\phi_d = 0.1$ rad. If the desired settling time of the response is 5 seconds, evaluate whether the response meets the desired specifications in terms of overshoot and settling time. [**M**]

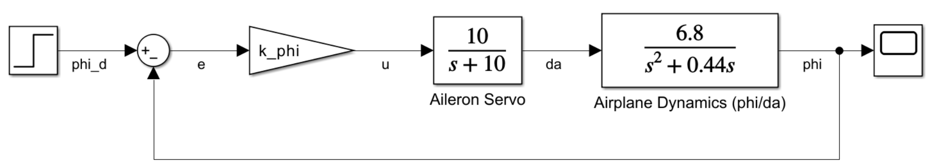

Figure 4: Block diagram for Bank Angle Hold Mode (Autopilot)

(b) Assume that the aileron servo has the general transfer function $G(s) = \frac{a}{s+a}$ where a is the servo bandwidth. Analyze the effect of bandwidth of servo on the speed of the closed-loop response by comparing the step response of closed-loop system for $a = 1, 10, 40$ and $k_\phi = 0.024$. You may use `feedback` command in MATLAB to find the closed-loop system. (Hint: Consider the setteling time for your comarison) [**M**]

(c) Based on the root locus analysis from part a, it is evident that we cannot simultaneously reduce the overshoot (by increasing the damping ratio) and enhance the speed of response (by increasing the frequency) through solely adjusting the proportional gain in a single angle control loop. To increase the system's damping ratio and enhance the closed-loop system response, we could include an inner loop (rate damper) and form a cascade control system as illustrated in Figure 5.

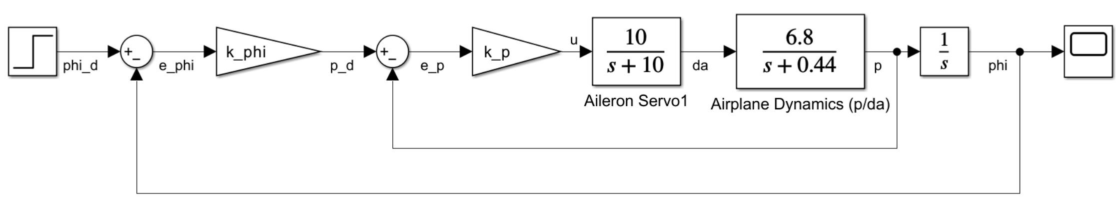

Figure 5: Block diagram for Bank Angle Hold Mode with Inner Loop Roll Rate Feedback

To derive the gains analytically based on the desired closed-loop system dominant poles, we can neglect the aileron servo transfer function and calculate the gains by equating the closed-loop system to a standard second-order system. Assuming the desired damping ratio and natural frequency are $\zeta = 0.7$ and $\omega_n = 5$ rad/s, determine the gains required to achieve the desired closed-loop system and simulate the control system in Simulink. Does the simulation result meet the specified requirements and demonstrate an acceptable response? Compare the results with those obtained in part a. [**M**]

(d) There are various methods for tuning the gains of multi-loop control systems. One practical approach is to tune the inner loop first and then proceed to tune the outer loops. This sequence ensures that the inner loop is functioning correctly before tuning the outer loop. However, it is crucial to guarantee the stability of the overall closed-loop control system.Assume we want to improve the performance of the inner loop in the previous control system (considering the aileron servo) by using a PID controller instead of just proportional gain. Design the PID gains of the inner loop controller using the aforementioned method to achieve the desired closed-loop (inner loop) response performance of 10% overshoot and settling time of 2 seconds. You may utilize the PID Tuner app (in the PID Controller block) or the Control System Designer app. Then, select the appropriate k_ϕ using either the root locus method (by determining the inner loop closed-loop transfer function) or by using the PID controller block (specifying the controller type as P and then using the PID Tuner). Afterwards, simulate the closed-loop system in Simulink and analyze the result. [**M**]

(e) A block diagram for a heading angle χ hold mode is depicted in Figure 6. The Bank Angle Autopilot subsystem includes the the previously designed bank angle two-loop closed-loop control system. Assume the airplane is in a cruise flight condition with a speed of $V = 205$ m/s and initial heading angle of $\chi_0 = -0.2$ rad. Design the k_χ gain by replacing it with the PID block and using the PID Tuner app to achieve a compensated system with a phase margin of 63 degrees and gain margin of 11.5 dB. Check the transient response characteristics and simulate the heading angle hold autopilot in Simulink to maintain the airplane heading angle at 0.2 rad. Set the saturation limit for the desired bank angle such that $|\phi_d| < 0.7$ rad (40 degrees). [**M**]

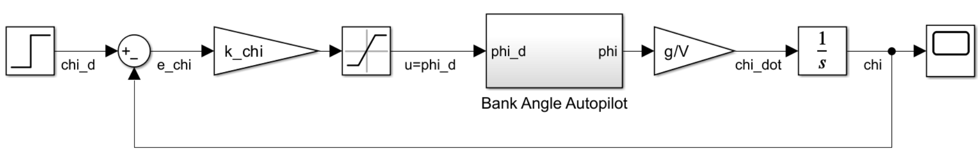

Figure 6: Block diagram for a Heading Angle Hold Mode (enclosing bank angle autopilot)

10. (a) (Bonus) Research and familiarize yourself with the techniques for automatic designing and tuning multi-loop control systems, such as employing optimization algorithms. Then, create a video to explain one of the techniques.

(b) (Bonus) Study the PID autotuner blocks in Simulink Control Design toolbox (not PID tuner app) and create a video to explain how they work and when should be used. You may refer to MATLAB Help documentation for more information.

Good Luck
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