In the Name of God

Amirkabir University of Technology
(Tehran Polytechnic)
Department of Electrical Engineering

Modern Control
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Fall 2024
Homework 2
Due: Oct 29, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner. You can also type the answers if you prefer.

- You may use MATLAB and Simulink or Python only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn how the method works and improve your problem-solving acumen by solving the problems by hand.

- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.

- Submit a compressed file with the naming format MC_HW1_FullName on the Courses platform. The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.

- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.

- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.

- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.

- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Consider the following system:

$$\dot{x} = \begin{bmatrix} -4 & 5 & -6 & -4 \\ -15 & 15 & -17 & -11 \\ 4 & -4 & 7 & 4 \\ -16 & 15 & -21 & -12 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

   Find the state transition matrix by using the Jordan canonical form of the system matrix.

2. Consider the following matrix:

$$A = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$$

   a) Apply the Cayley-Hamilton theorem for matrix A to derive the matrix function A^K.

   b) Show that the relation $\sin^2(A) + \cos^2(A) = I$ holds. Afterwards, check your results using the `funm` command in MATLAB. [**M**]

3. Consider the following MIMO system in state-space form:

$$\dot{x} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x$$

   a) Derive the system's transfer function matrix.

   b) Calculate the system's poles, transmission zeros, and McMillan's degree.

4. The system response with state-space equations is given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = Ax + bu, \quad y = Cx + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

   The state transition matrix for this system is as follows:

$$\phi(t) = \begin{bmatrix} e^{-t} & 0 \\ e^{-t} - e^{-2t} & e^{-2t} \end{bmatrix}.$$

   Additionally, the system's response is described as follows:

   - The system's response at $t = 3$ to an initial condition of $x(0) = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ is:

$$y(3) = \begin{bmatrix} 3e^{-3} - 2e^{-6} \\ e^{-6} \end{bmatrix}$$

- In addition, in the case of a step input with unknown initial conditions, the system state at $t = 1$ is $x(1) = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$ and the output at $t = 3$ is:

$$y(3) = \begin{bmatrix} 1 \\ 1 + 5e^{-2} \end{bmatrix}$$

Find the transfer function for this system in the following cases:

a) By calculating the matrix A.

b) Without calculating the matrix A.

5. The following diagram illustrates an underwater system.

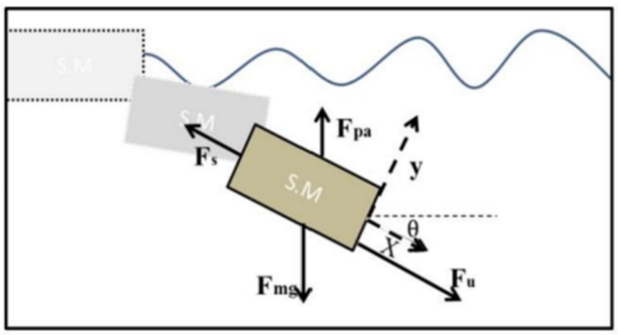

Figure 1: Underwater system

Based on the following assumptions:

- The resisting force acting on the motion in water is governed by the Square Law Drag.
- The underwater body's mass is denoted as m_1, and the equivalent mass of displaced water corresponds to m_2.
- The input signal is the force in the direction of x, while the system's output is x.

the differential equations for a nonlinear underwater slope motion, where the slope (m) changes are proportional to $x^2 - x$, are given as follows:

$$m_1\ddot{x} + F_s + \sin(\theta)(F_{pa} - F_{mg}) = F_u$$

$$F_s = K\dot{x}|\dot{x}| \quad \text{(square law)}$$

$$F_{pa} = \rho g V = m_2 g, \quad F_{mg} = m_1 g, \quad F_u = u$$

$$m = \tan(\theta), \quad \dot{m} = x^2 - x$$

a) Find the nonlinear state equations $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$.

b) Linearize the system around the following equilibrium point and obtain the state space representation of the system.

$$\mathbf{x}_e = \begin{bmatrix} x & \dot{x} & \theta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2k\pi \end{bmatrix}, \quad k = 0, 1, 2, \ldots$$

c) Linearize the system and obtain the state space model of the system utilizing MATLAB (Symbolic Math Toolbox). Compare the obtained state-space model with b. [**M**]

6. Consider the following system of interconnected inverted pendulums:

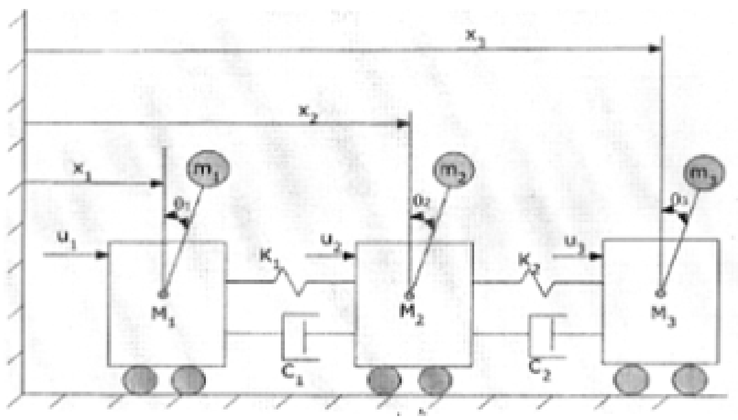

Figure 2: system of interconnected inverted pendulums

Additionally, the parameters of the system are provided in Table 1.

Table 1: Parameters of the system

Parameter	Value
M	2 kg
m	0.21 kg
L	0.21 m
c_1, c_2	0.06 N.s/m
k_1, k_2	1 N/m

a) Determine the degrees of freedom of the system and derive the system's dynamic equations using the Lagrange Method.

b) Linearize the system around the equilibrium point and find the state space representation for the system. [**M**]

c) (Bonus) Obtain the zero-input response of the system to an arbitrary initial condition. [**M**]

d) (Bonus) Write a MATLAB function that takes an input matrix of the system state data and plots the animation of motion for the interconnected system. [**M**]

Good Luck
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