In the Name of God

Amirkabir University of Technology
(Tehran Polytechnic)
Department of Electrical Engineering

Modern Control
Dr. H. Atrianfar
Fall 2024
Homework 4
Due: Nov 20, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner. You can also type the answers if you prefer.
- You may use MATLAB and Simulink or Python only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn how the method works and improve your problem-solving acumen by solving the problems by hand.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format MC_HW1_FullName on the Courses platform. The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Prove that if an LTI system in state-space form is asymptotically stable, then it is also BIBO (Bounded Input, Bounded Output) stable. (Hint: Assume the inputs and initial states are bounded with limits a and b, respectively, and show that the output is also bounded, depending on a and b.)

2. Consider the following LTI system:

$$\dot{x} = \begin{bmatrix} -3 & 4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} x$$

   a) Analyze the BIBO stability and internal stability of the system.
   b) If the system is not BIBO stable, suggest a new control effectiveness (matrix B) such that the new system becomes BIBO stable.

3. Utilize the Lyapunov stability theorem to find the range of k for which the system is asymptotically stable. Is the system globally asymptotically stable?

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k & -5 & -6 \end{bmatrix} x$$

   The candidate Lyapunov function $V(x)$ is given by:

$$V(x) = 6kx_1^2 + 2kx_1x_2 + 41x_2^2 + 12x_2x_3 + x_3^2$$

4. Using the Lyapunov theorem, obtain the sufficient conditions for the stability of a satellite with the following equations:

$$\begin{cases} A\dot{\omega}_x - (B - C)\omega_y\omega_z = k_1 A\omega_x \\ B\dot{\omega}_y - (C - A)\omega_z\omega_x = k_2 B\omega_y \\ C\dot{\omega}_z - (A - B)\omega_x\omega_y = k_3 C\omega_z \end{cases}$$

5. Consider the following system:

$$\begin{cases} \dot{x}_1 = x_1^2 x_2 + x_1 x_2^2 \\ \dot{x}_2 = (-x_1 + x_2)(x_2^2 - 1) \end{cases} \tag{1}$$

   a) Find the equilibrium points and specify their type. (Hint: Linearize the system around the equilibrium points)
   b) (Bonus) Sketch the phase portrait for each system and analyze the stability of the equilibrium points and determine their types. Compare your result with part a. You may utilize the `plotpp` provided in the HW file. [**M**]

6. A moon lander (lunar lander) is illustrated below.

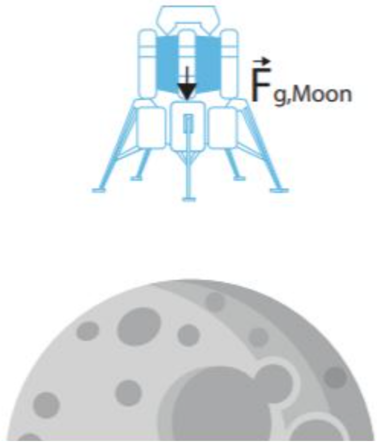

Figure 1: Schematic of a moon lander

A simplified dynamics of this system can be represented as follows:

$$\ddot{h} = -g + u$$

where the h and u denote the height and control force (thrust), respectively.

a) The mission requires that the lander track a desired height trajectory with the following profile to achieve a smooth landing.

$$h_d(t) = (100 - t)e^{-t/20}$$

Design a control law u based on the Lyapunov theorem to ensure that the lander follows this trajectory with zero steady-state error. (Hint: Use the candidate Lyapunov function $V(s) = 0.5s^2$ where $s = \dot{e} + \lambda e$, $e = h_d - h$ and λ is a design parameter.)

b) Assuming $g_{moon} = 1.62\ m.s^{-2}$, $h_0 = 90$ m and $\dot{h}_0 = 0$, simulate the closed-loop system for 100 seconds and evaluate the effectiveness of the designed control law. Tune the parameters to ensure that the tracking error tend to zero within 20 seconds. Discuss how the parameters k and λ influence the closed-loop system response. [**M**]

c) If we impose a constraint whereby the control signal must remain positive (due to thruster limitations), apply a saturation constraint with limits of 0 to 5 on the control signal, and analyze the system's response under these more realistic conditions. [**M**]

Good Luck
M. Shahrajabian