In the Name of God

Amirkabir University of Technology
(Tehran Polytechnic)
Department of Electrical Engineering

Modern Control
Dr. H. Atrianfar
Fall 2024
Homework 5
Due: Dec 13, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner. You can also type the answers if you prefer.
- You may use MATLAB and Simulink or Python only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn how the method works and improve your problem-solving acumen by solving the problems by hand.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format MC_HW1_FullName on the Courses platform. The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. Consider the following transfer function:

$$g(s) = \frac{s - 1 + a}{s^3 + (2+b)s^2 - s - (2+b)}$$

   (a) Find the controller canonical form for this system and analyze the observability for various values of a and b.

   (b) Find the observer canonical form for this system and analyze the controllability for various values of a and b.

2. Consider the following two systems:

$$S_1: \quad A_1 = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} \alpha & 1 \end{bmatrix}$$

$$S_2: \quad A_2 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_2 = \begin{bmatrix} \beta & 1 \end{bmatrix}$$

   where $\alpha, \beta \in \mathbb{R}$.

   (a) For what values of α and β does a similarity transformation between the two systems exist?

   (b) For a given pair of values α and β for which a similarity transformation exists, find the transformation and verify your answer.

3. For the system below, determine the controller canonical, controllability canonical, observer canonical, and observability canonical forms. Then analyze their controllability and observability.

$$G(s) = \frac{s^3 + 4.5s^2 + 5s + 1.5}{s^4 + 8.5s^3 + 26s^2 + 33.5s + 15}$$

4. A single-input single-output system with the following transfer function is given:

$$H(s) = \frac{(s+5)(s+\alpha)}{(s+\alpha)(s+2)(s+3)}$$

   a) Obtain the differential equation of the system using pole-zero cancellation and by sketching the block diagram.

   b) Do the previous part without using pole-zero cancellation and with the canonical state-space representation.

   c) Compare the controllability and observability of the systems achieved in the two parts above.

5. (a) Find a state space representation for the following system:

$$H(s) = \begin{bmatrix} \frac{3}{(s^2+2s+10)(s+5)} & \frac{2s}{(s+5)} \\ \frac{1}{s+1} & \frac{(s+5)}{s+1} \end{bmatrix}$$

   (b) Analyze the irreducibility of the system.

6. The system H is formed as a combination of the following three subsystems:

$$S_1 = \frac{s-1}{s^2+2s+2}, \quad S_2 = \frac{1}{s^2+s-2}, \quad S_3 = \frac{s+2}{s^2+4s+4}.$$

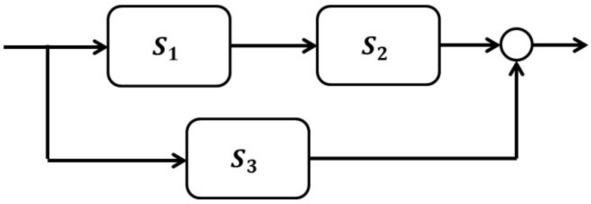

Figure 1: Block diagram of the system H

Find the state-space representation of the overall system.

7. (Bonus) Implement an algorithm to obtain Markov parameters from a transfer function. The code should prompt the user to input the coefficients of the numerator and denominator of the transfer function. The output should be the Markov parameters up to the order $2n-1$, where n is the degree of the denominator. Notice that you should find the parameters directly from the transfer function. [**M**]

Good Luck
M. Shahrajabian