**In the Name of God**

Amirkabir University of Technology
(Tehran Polytechnic)
**Department of Electrical Engineering**

**Modern Control**
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**Homework 6**
**Due: Dec 24, 2024**

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**Please notice the following:**

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner. You can also type the answers if you prefer.
- You may use MATLAB and Simulink or Python only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first.* You will not gain any intuition by looking at results — you need to learn how the method works and improve your problem-solving acumen by solving the problems by hand.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format MC_HW1_FullName on the Courses platform. The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually.* While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

## Questions

1. Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} x.$$

   (a) Show that the system is uncontrollable and determine its uncontrollable modes.

   (b) Consider the state feedback of the form $u = -[k_1\ k_2]x + u_{\text{ext}}$. Obtain the closed-loop state-space equations.

   (c) Show that the system obtained in part (b) retains the uncontrollable modes corresponding to the parameters $[k_1\ k_2]$. Compare the system in this case with part (a).

   (d) Derive the transfer function $\frac{y}{u_{\text{ext}}}$ and show that by selecting suitable values for $[c_1\ c_2]$ and $[k_1\ k_2]$, the system has no zeros and only one pole at the origin.

2. Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -1 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 10 & 1 & 0 \end{bmatrix} x$$

   (a) Consider a state feedback of the form $u = -kx$. Find the eigenvalues of the closed-loop system.

   (b) Design the state feedback such that the closed-loop system response has a settling time of less than 4 seconds and an overshoot of less than 15%. Determine the desired closed-loop poles.

   (c) If the control law is in the form $u = -kx + r$, calculate the gain $k$ for the closed-loop system using the Ackermann's formula and the canonical form methods. Compare your result with those from the `place` command.

   (d) Simulate the closed-loop response with the obtained gains and verify your design. [**M**]

   (e) Design the the static pre-compensator for the system such that the steady-state error to the step input is zero. [**M**]

   (f) Design a dynamic pre-compensator such that the system has zero steady-state error for a step input. Assume the closed-loop poles are at $\{-4, -2, -1 \pm i\}$. [**M**]

   (g) Add a constant disturbance to the control input and compare the performance of the designed controllers in parts (f) and (g) in the tracking step reference scenario. [**M**]

3. Consider the following multi-input system:

$$A = \begin{bmatrix} -4 & 11 & 30 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

   Find the state feedback gain $k$ such that the closed-loop poles are placed at $\{-2, -1 \pm i\}$. Find the gain $k$ using the `place` command. Are the gains equal? Explain!

4. Design a reduced-order observer for the following system such that the eigenvalues of the observer are located at $-1$ and $-2$.

$$\dot{x} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} x, \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x.$$

5. Consider the following system:

$$G(s) = \frac{1}{s(s^2-1)}.$$

   (a) Design a state feedback controller for the system such that the closed-loop poles are located at $s = \{-1.5 \pm i, -2\}$.

   (b) Design a full-order observer and a reduced-order observer for this system. Once, place the observer poles at $-0.05$; and then place them at $-10$.

   (c) Simulate the closed-loop response of the system using the designed state feedback controller and observers in part (b). Compare the performance of the observers in estimation. Which observer performs best in estimation and results in better regulation for control? [**M**]

6. The linearized equations of motion for an inverted pendulum are given as follows (state variables are considered as $x_1 = x, x_2 = \dot{x}, x_3 = \theta, x_4 = \dot{\theta}$):

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{m_1 m_2 g}{1-m_1 m_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{m_2 g}{1-m_1 m_2} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{(m+M)(1-m_1 m_2)} \\ 0 \\ -\frac{m_2}{(m+M)(1-m_1 m_2)} \end{bmatrix} u(t)$$

$$\text{where} \quad m_1 = \frac{ml}{M+m}, \quad m_2 = \frac{ml}{I+ml^2}$$

The physical parameters of the system are described in the table below.

Table 1: System Parameters

| Parameter | Value |
|---|---|
| $M$ | 0.5 kg |
| $m$ | 0.05 kg |
| $g$ | 9.8 m/s$^2$ |
| $l$ | 0.3 m |
| $I$ | 0.006 kg/m$^2$ |

   (a) Assume that $x_1$ and $x_3$ are measurable; and design a reduced-order observer such that its poles are located at $-10$.

   (b) (Bonus) Design a dynamic pre-compensator using the LQR method to regulate the system to the desired states. Simulate the closed-loop system with the estimated states, the initial condition of $x_0 = [0.5,\ -0.1,\ 0.1,\ 0.05]^T$ and desired state of $x_d = [0,\ 0,\ -0.1,\ 0]$ [**M**].

**Good Luck**

**M. Shahrajabian**